Intersymbol Interference for Electrophoretic Molecular Communication in Circular Duct Channels

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***Abstract*—This paper investigates electrophoretic molecular communication (EMC) operating in circular duct channels. EMC utilizes the time-varying electrophoretic force that can controllably induce the movement of charged particles to enhance communication performance. In circular duct channels, where the memory component is high, intersymbol interference (ISI) must be considered. Thus, this paper presents a method to design an electric field under the framework of the calculus of variations, simultaneously reducing the ISI and strengthening the information signal reception. The numerical results show that the proposed electric field can significantly reduce the bit error rate compared to the constant field benchmark.**

***Index Terms*—Molecular communication, electrophoresis, nanonetworks, intersymbol interference**

I. Introduction

The development of nanotechnology enables the miniaturization and fabrication of devices in a nano- and microcopic size scale ranging from 1 to 100 nanometers. As a basic functional unit, a nano-machine can perform elementary and specific tasks, such as sensing, actuation, computing, and data storage [1]. Such nano-machines have to communicate with each other and constitute a nanonetwork to coordinate, share, and fuse information to expand and overcome their limited capability. The traditional radio frequency (RF) communication technologies are impractical at such small dimensions because, for example, the antenna size for RF communication must be the scale of the signal's wavelength, which is not feasible in nano- or micro-scale. Fortunately, molecular communication (MC), which utilizes molecules as biochemical signals to encode, transmit and receive information, can be a solution for nanoscale information exchange due to various advantages of microminiature transceivers, low energy consumption, biocompatibility, etc. [1].

Compared to the RF communications, the memory component of MC channels is very high, since the information-carrying molecules have a probabilistic movement pattern that is subject to Brownian motion. This extended memory gives rise to intersymbol interference (ISI), which causes a signal distortion in which one symbol interferes with subsequent symbols. In order to mitigate the ISI effect, an advection can be considered to move the information-carrying molecules quickly from a transmitter to a receiver before the molecules spread widely [2]. Moreover, various communication techniques for ISI mitigation, tailored for MC systems, such as ISI cancellation based on the previous bit information, decoder threshold optimization, channel coding, etc. [3]–[5], have been proposed and investigated.

Our recent study [6] proposed utilizing a time-varying electrophoretic force, which induces the movement of dispersed particles relative to a fluid under the influence of a spatially uniform electric field, to propagate charged information-carrying molecules in MC systems controllably. The work revealed that electrophoresis significantly improves MC performance, especially lowering the bit error rate (BER), by increasing the number of observed molecules during the intended symbol period while reducing the number of residual molecules afterward. Please refer to [6] for more details on possible applications and advantages of utilizing time-varying electrophoretic force in MC systems.

On the other hand, the work in [6] has a limitation that it assumed an unbounded channel environment, in which the ISI may not be considerable because there is enough room for the molecules to diffuse away after being counted at the receiver. Hence, this paper investigates exploiting an electrophoretic force in a bounded circular duct channel, in which the ISI will be more significant. This work explains the impact of the electrophoretic force on the expected number of observed molecules in a circular duct channel, then proposes an optimization technique to design the electric field. The optimization problem retains the cost function that reflects both the information signal in a current bit interval and the ISI component in the subsequent bit interval. Employing the cost function provides a method of balancing the information signal and interference strengths to improve the MC performance. We show that the proposed field significantly lowers the BER relative to constant advection by effectively coping with the ISI influence.

II. System Model

A. Transmitter, Receiver, and Flow

A straight, impermeable, cylindrical duct channel of infinite axial extent and radius a_c is considered. We utilize the cylindrical coordinates (r, θ, x), where $r \in [0, a_c]$ is the radial distance, $\theta \in (0, 2\pi]$ is the azimuth angle, and $x \in (-\infty, \infty)$ is the axial position. The duct channel is assumed to be filled with neutrally charged fluid.

The receiver is a transparent and spherical passive observer with radius r_{obs}, fixed and centered at the origin, i.e., $(0, 0, 0)$. The receiver is assumed to detect and count the particles

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This work was made possible by the OUP's John Fell Fund (Grant No. 0006235). AACP and TCS were also funded by a Royal Society University Research Fellowship (Grant No. URF\R\180016) and Enhancement Award (Grant No. RGF\EA\181002).

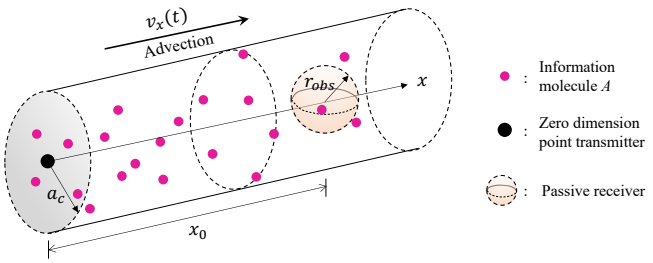

Fig. 1. Schematic description of the system model.

within its volume and not impede diffusion or initiate chemical reactions. This configuration corresponds to passive detection by, for example, optical biosensors [7], though other types of receiver may see information-carrying molecules absorbed. We consider a dimensionless point transmitter, which is a source of information type-A molecules, and is fixed at location $\mathbf{d}_{\text{TX}} = (0, 0, -x_0)$. The transmitter has a B-bit binary sequence $\mathbf{W} = \{W[1], W[2], \ldots, W[B]\}$ to send to the receiver, where $W[j]$ is the jth information bit and $\Pr(W[j] = 1) = P_1$. The transmitter uses binary modulation and transmission intervals of duration T_{int} seconds. To send a binary 1, N_{EM} molecules are released in an impulsive manner at the start of the bit interval. To send a binary 0, no molecules are released.

To initiate EMC, we apply a time-varying electric field $\mathbf{E}(t)$ uniformly over the entire channel. This field induces an electrophoretic force $\mathbf{F}_E(t) = q_A\mathbf{E}(t)$, which produces a flow of type-A molecules with time-varying velocity $\mathbf{v}(t)$, where q_A denotes the electric charge on a single type-A molecule. We assume that the molecule velocity is linearly related to the electrophoretic force, i.e., $\mathbf{v}(t) \propto \mathbf{E}(t)$. $\mathbf{v}(t)$ is defined by its velocity component along each dimension, i.e. $\mathbf{v}(t) = (v_r(t), v_\theta(t), v_x(t))$, while we assume a flow only in the x direction, so only the vector component $v_x(t)$ is considered in this paper. The electric field is periodically applied regardless of which bit is transmitted.

B. Receiver Signal

For a given time t, $C_A(\mathbf{d}, t; t_0)$ denotes the concentration of type-A molecules in molecules·m^{-3} at a point $\mathbf{d}$, that were released from the transmitter at time t_0 (or written as C_A for compactness). We assume that these molecules independently travel once they are released, either by the transmitter or sources of noise, with no electrostatic repulsion [6]. In addition, assuming the environment has a constant uniform temperature and viscosity, the type-A molecules diffuse with a constant diffusion coefficient D_A (m^2/s). Fick's second law describes the motion of type-A molecules due to both advection and diffusion by the differential equation as

$$\frac{\partial C_A}{\partial t} + \mathbf{v}(t) \cdot \nabla C_A = D_A \nabla^2 C_A. \tag{1}$$

For a circular duct channel with fully reflective walls, C_A at point $\mathbf{d} = (r, \theta, x)$ can be derived from (1) by using a moving reference frame with the following IC and BCs:

$$\text{IC} : C_A(\mathbf{d}, t_0; t_0) = N_{\text{EM}}\delta(\mathbf{d} - \mathbf{d}_{\text{TX}}), \tag{2a}$$

$$\text{BC}_1 : \left.\frac{\partial C_A(\mathbf{d}, t)}{\partial r}\right|_{r=a_c} = 0, \tag{2b}$$

$$\text{BC}_2 : C_A(\mathbf{d} = (r, \theta, x \to \infty), t; t_0) = 0 \tag{2c}$$

as

$$C_A(\mathbf{d}, t; t_0) = \frac{N_{\text{EM}} \exp\left(\frac{-\left(x + x_0 - \int_{t_0}^{t} v_x(t)\,\mathrm{d}t\right)^2}{4D_A(t - t_0)}\right)}{2\pi a_c^2\sqrt{\pi D_A(t - t_0)}} \times \tag{3}$$
$$\left[1 + \sum_{n=-\infty}^{+\infty}\left\{\cos(n\theta)\sum_{\alpha} e^{-D_A\alpha^2(t-t_0)}\frac{\alpha^2 J_n(\alpha r)J_n(0)}{(\alpha^2 - n^2/a_c^2)J_n^2(a_c\alpha)}\right\}\right]$$

where the summation in α is over the positive roots of $J_n'(\alpha a_c) = 0$ [8]. $J_n(\cdot)$ denotes the nth order Bessel function of the first kind and $J_n'(\cdot)$ denotes its derivative. Please refer to [8] for further details on the derivation.

The receiver is a passive observer, so the expected number of type-A molecules within the receiver volume (due to a single emission of molecules) is found by integrating (3) over the volume of the receiver sphere $\mathcal{V}_{\text{obs}}$, i.e.,

$$\overline{N_{A_0}}(t; t_0) = \iiint_{\mathbf{d}\in\mathcal{V}_{\text{obs}}} C_A(\mathbf{d}, t; t_0) r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}x. \tag{4}$$

This integral can be simplified by utilizing the uniform concentration assumption [9], where the expected concentration throughout the receiver is assumed to be equal to that expected at the center of the receiver, to

$$\overline{N_{A_0}}(t; t_0) = V_{\text{obs}} C_A(\mathbf{d}_{\text{RX}}, t; t_0) \tag{5}$$
$$= V_{\text{obs}} \frac{N_{\text{EM}} \exp\left(\frac{-\left(x_0 - \int_{t_0}^{t} v_x(t)dt\right)^2}{4D_A(t-t_0)}\right)}{2\pi a_c^2\sqrt{\pi D_A(t - t_0)}}\left[1 + \sum_{\alpha}\frac{e^{(-D_A\alpha^2(t-t_0))}}{J_0^2(a_c\alpha)}\right]$$

where $\mathbf{d}_{\text{RX}} = (0, 0, 0)$ denotes the receiver's center, i.e., the origin of the considered cylindrical coordinates, and the summation in α is over the positive roots of $J_0'(\alpha a_c) = 0$.

The statistics of the general receiver signal $N_{A_{\text{obs}}}(t)$, i.e., the number of observed molecules due to sequential transmissions and noise, can be derived based on $\overline{N_{A_0}}(t; t_0)$ and the transmitter sequence $\mathbf{W}$. Assuming that the behavior of individual type-A molecules is independent, $N_{A_{\text{obs}}}(t)$ is a sum of time-varying Poisson random variables as shown in [10], with time-varying mean

$$\overline{N_{A_{\text{obs}}}}(t) = \overline{N_{A_{\text{TX}}}}(t) + \overline{N_{A_n}}(t) \tag{6}$$

where $\overline{N_{A_n}}(t)$ is the mean number of molecules from the noise sources, and $\overline{N_{A_{\text{TX}}}}(t)$ is the mean number of observed molecules due to sequential emissions by the transmitter, i.e.,

$$\overline{N_{A_{\text{TX}}}}(t) = \sum_{j=1}^{\lfloor \frac{t}{T_{\text{int}}}+1 \rfloor} W[j]\overline{N_{A_0}}\left(t; (j-1)T_{\text{int}}\right). \tag{7}$$

In addition, we utilize the weighted sum detector as in [6] with a constant sampling interval t_s and M samples in every bit interval; thus, the global sampling time function can be expressed as $t(j, m) = jT_{\text{int}} + mt_s$, where $j = \{1, 2, \ldots, B\}$, $m = \{1, 2, \ldots, M\}$.

III. Electric Field Design

This section investigates designing the electric field to improve the bit error performance of the proposed electrophoretic MC system in a circular duct channel, relative to benchmarks. In MC systems, the information-carrying molecules emitted at the beginning of the current bit interval should arrive at the receiver site as quickly as possible. This creates a high molecule concentration at the receiver site, and ensures a strong information signal. Note that the molecules spread widely as time elapses, i.e., the concentration decreases. After being counted by the receiver, the molecules should move away to not interfere with the information transmission in the next bit interval. Therefore, it would be sensible to define the cost function to minimize the deviation of the center of the molecule group from the receiver site during the current bit interval, i.e., $t \in [0, T_{\text{int}}]$, while increasing the deviation during the next bit interval, i.e., $t \in [T_{\text{int}}, 2T_{\text{int}}]$. Here, we use the center location of the molecule group to design the electric field, since the molecules randomly diffuse after being injected into the channel, and the molecules' density is likely to be the highest at the center of the molecule group.

Hence, we define the cost function as

$$C[x(t), x'(t)] = \frac{1}{T_{\text{int}}} \left(\int_0^{T_{\text{int}}} (x(t) - x_0)^2 \, \mathrm{d}t - q \int_{T_{\text{int}}}^{2T_{\text{int}}} (x(t) - x_0)^2 \, \mathrm{d}t \right) \tag{8a}$$

$$\text{s. t.} \begin{cases} \dfrac{1}{T_{\text{int}}} \displaystyle\int_0^{T_{\text{int}}} v_x^2(t) dt \le \xi_v, \\ IC : x(0) = 0, \\ FC : x(T_{\text{int}}) = x_1 \end{cases} \tag{8b}$$

where $x(t) = \int_0^t v_x(t) \, \mathrm{d}t$ denotes the travel distance of the center of the molecule group from the transmitter site, and $x'(t) = v_x(t)$. Here, the first integration term in (8a) corresponds to the mean square error (MSE) between the center of the molecule group and the receiver site during the current bit interval, i.e., which needs to be minimized, while the second term corresponds to the MSE during the next bit interval, i.e., which needs to be maximized; hence, the sign of the second term is negative. Also, $q \ge 0$ denotes the weight parameter that we set, which corresponds to modifying the electric field strength over time. Also, this work assumes a system environment that the ISI component only in the first memory slot is significant; hence, the second term in (8) considers only the time interval $t \in [T_{\text{int}}, 2T_{\text{int}}]$. The first constraint in (8b) constrains the average power of the applied electric field to ξ_v. Note that since $v_x(t)$ is the flow velocity of the molecules linearly induced by $\mathbf{E}(t)$, constraining the mean square of the flow velocity is equivalent to constraining the average power of the electric field. The initial and final conditions are set as 0 and x_1, respectively, where the final condition means that the center of the molecule group should travel the distance of x_1 when the current bit interval finishes. The final condition would be conditioned to the requirements of designing an LoC application, e.g., the location of transducer/biosensor outlets.

Since the electric field is periodic for every bit interval, i.e., $x(t + T_{\text{int}}) = x(t) + x_1$, the cost function can be rewritten as

$$C[x(t), x'(t)] = \frac{1}{T_{\text{int}}} \int_0^{T_{\text{int}}} (x(t) - x_0)^2 - q \, (x(t) + x_1 - x_0)^2 \, \mathrm{d}t. \tag{9}$$

Then, we form the Lagrangian as

$$\mathcal{L}[x(t), x'(t)] = [x(t) - x_0]^2 - q[x(t) + x_1 - x_0]^2 + \mu[x'(t)]^2 \tag{10}$$

where $\mu \in \mathbb{R}$ denotes the Lagrangian multiplier. From (10), the Euler–Lagrange equation [11] can be formulated as

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial \mathcal{L}}{\partial x'(t)} \right) - \frac{\partial \mathcal{L}}{\partial x(t)} = \frac{\mathrm{d}}{\mathrm{d}t} \left(2\mu x'(t) \right) - 2 \left[x(t) - x_0 \right] + 2q \left[x(t) + x_1 - x_0 \right] = 0. \tag{11}$$

This formulation can be further simplified to

$$x''(t) - A(\mu) x(t) + B(\mu) = 0 \tag{12}$$

where $A(\mu) = (1 - q)/\mu$ and $B(\mu) = ((1 - q)x_0 + qx_1)/\mu$. The solution of the differential equation (12) is

$$x(t) = C_1 e^{\sqrt{A(\mu)} t} + C_2 e^{-\sqrt{A(\mu)} t} + K \tag{13}$$

where $K = B(\mu)/A(\mu)$. C_1 and C_2 are given by

$$C_1 = \frac{(e^{-\sqrt{A(\mu)} T_{\text{int}}} - 1)K + x_1}{e^{\sqrt{A(\mu)} T_{\text{int}}} - e^{-\sqrt{A(\mu)} T_{\text{int}}}} \tag{14a}$$

$$C_2 = -\frac{(e^{-\sqrt{A(\mu)} T_{\text{int}}} - 1)K + x_1}{e^{\sqrt{A(\mu)} T_{\text{int}}} - e^{-\sqrt{A(\mu)} T_{\text{int}}}} - K. \tag{14b}$$

μ can be numerically found from the first constraint in (8b):

$$\int_0^{T_{\text{int}}} (x'(t))^2 dt = \frac{C_1^2 \sqrt{A(\mu)}}{2} (e^{2\sqrt{A(\mu)} T_{\text{int}}} - 1) - \frac{C_2^2 \sqrt{A(\mu)}}{2} (e^{-2\sqrt{A(\mu)} T_{\text{int}}} - 1) - 2A(\mu) C_1 C_2 T_{\text{int}} = \xi_v T_{\text{int}}. \tag{15}$$

Fig. 2 shows the travel distance of the center of the molecule group $x(t)$ and the x-axis velocity of the molecules $v_x(t)$, respectively, for different final conditions x_1 and different values of the weight parameter q. The other system parameters utilized in Section IV are adopted. The designed velocities start with a very high value at the beginning of the bit interval to accelerate the molecules just injected into the channel toward the receiver location. Then, the velocity gradually decreases to let the molecules stay around the receiver site. Again, the velocity increases just before the current bit interval ends to propagate the molecules away not to interfere with the molecules in the next bit interval. Compared to the constant field that retains the identical average power of the proposed fields, the paths for the proposed fields in Fig. 2 reach the receiver site more quickly and stay there for a longer time, which would increase the number of molecules observed at the receiver site. The second graph in Fig. 2 shows how q changes the applied electric field (compare the like-coloured lines), recalling the bijective relationship between $\mathbf{v}(t)$ and $\mathbf{E}(t)$ introduced in Section II. Setting $q = 0.3$ takes the center of the molecule group off the receiver site during the middle

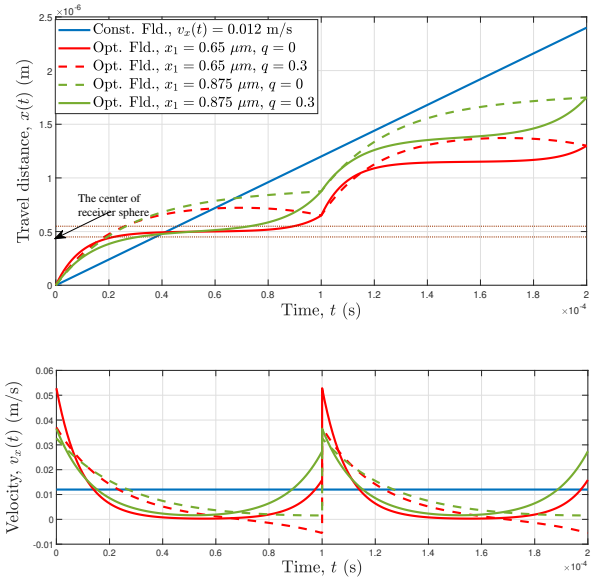

Fig. 2. The travel distance of the center of the molecule group $x(t)$ and the x-axis velocity $v_x(t)$ of the molecules according to the elapsed time, when the proposed electric field obtained from (8) is applied. $\xi_v = 1.44 \times 10^{-4}$ is assumed.

of the bit interval, compared to that of $q = 0$, (e.g., see the dashed lines for the period of $0.2 \sim 0.8 \times 10^{-4}$ s), which would reduce the ISI in the next bit interval.

IV. Numerical Results

This section provides numerical results to verify the benefits of the proposed electrophoretic forces in improving the MC performance in bounded circular ducts. The considered system parameters are $N_{\text{EM}} = 300$, $P_1 = 0.5$, $B = 100$ bits, T_{int} =0.1 ms, $D_A = 10^{-9}$ m^2/s, x_0= 0.5 μm, $r_{\text{obs}} = 50$ nm, $M = 5$, $\xi_v = 1.44 \times 10^{-4}$, and $\overline{N_{A_n}}(t) = 1$ molecule, which are adopted from [6]. The cylindrical duct radius $a_c = x_0/10$ is utilized. On the other hand, in the simulation for obtaining the BER performance, we generate Poisson random variables with the time-varying mean provided in (6) to mimic the observation value and estimate the transmitted bit $W[j]$. The simulation results are obtained by averaging over 3×10^3 independent Monte Carlo trials.

Fig. 3 shows the average number of observed molecules $\overline{N_{A_{\text{obs}}}}(t)$ when variations of the proposed time-varying electric field, corresponding to different values of the weight parameter q, are applied. Compared to the no field (i.e. diffusion dominated) and constant field cases, the proposed electric fields yield a higher signal strength, i.e., higher average number of observed molecules during the first bit interval $0 \sim 10^{-4}$ s. At the same time, the ISI of the proposed fields in the next time interval $10^{-4} \sim 2 \times 10^{-4}$ s also appears higher than that of the constant field. These results come from the design of the proposed fields, which rapidly sends the information molecules toward the receiver site right after they are injected into the channel, and make them stay there as long as possible. Consequently, the BER performance for the constant field and the time-varying proposed fields with $q = \{0, 0.1, 0.2, 0.3\}$

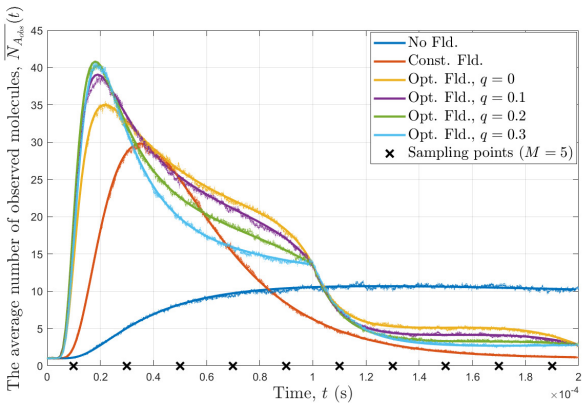

Fig. 3. The average number of molecules observed in the receiver $\overline{N_{A_{\text{obs}}}}(t)$ for the time-varying electrophoretic forces with different values of q as time elapses. $\mathbf{W} = \{1, 0, ...\}$ is assumed to be transmitted, and only the first two bits are described. The solid lines are obtained from numerically evaluating (6), while the dotted lines are obtained by averaging over 10^3 independent particle-based simulation trials. The final condition $x_1 = 0.875$ μm is used.

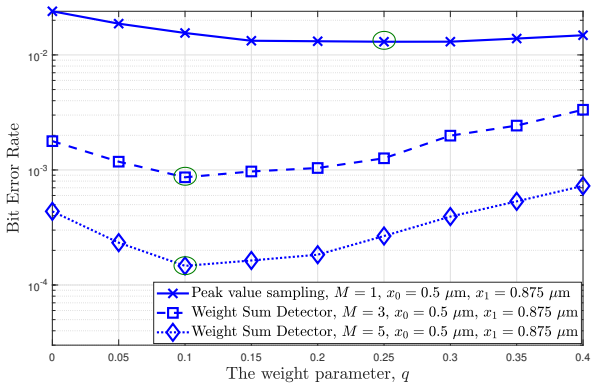

Fig. 4. The BER according to the weight parameter q. The peak value sampling method counts the number of molecules at the time that the *average* number of observed molecules $\overline{N_{A_{\text{obs}}}}(t)$ becomes the highest (e.g., in Fig. 3, the proposed fields have the peak value at around 0.2×10^{-4}.).

are given by $\{0.481, 0.452, 0.166, 0.185, 0.434\} \times 10^{-3}$, respectively, which verifies that the proposed fields provide a significant BER improvement, compared to the constant and no field cases.

As seen in Fig 3, increasing the weight parameter q can reduce the amount of ISI in the second bit interval; however, it also weakens the signal strength in the first interval (see the region from 0.3×10^{-4} to 1×10^{-4}). This observation is natural due to the periodicity of the electric field. In communication theory, the BER performance is affected by both the information signal strength and ISI; thus, a decent value of q needs to be chosen to achieve a satisfactory BER performance. In Fig. 4, the BER decreases as q increases at first but starts to increase as q continues to grow. This is because a large value of q sacrifices the signal strength too much, which worsens the BER performance despite lower ISI. In addition, it is noted that the values of q minimizing the BER (highlighted by the green circles) are different according to the considered system parameters, which implies that the appropriate q value varies

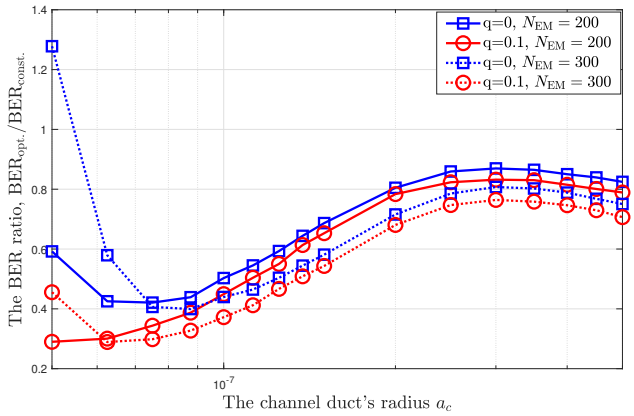

Fig. 5. The ratio of the BERs obtained from adopting the proposed field and the constant field (i.e., $BER_{opt.}/BER_{const.}$) according to the different channel radius a_c. $x_1 = 0.875$ μm is used.

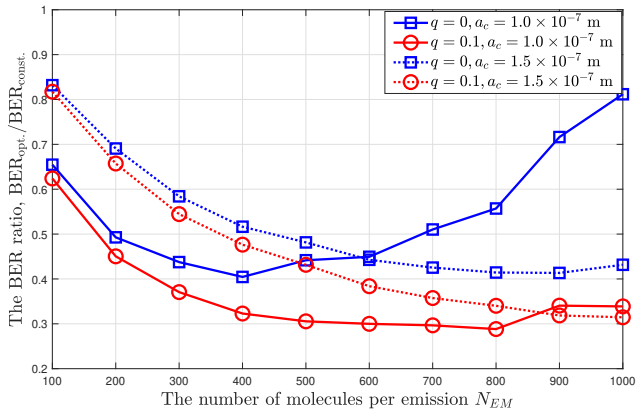

Fig. 6. The ratio of the BERs obtained from adopting the proposed field and the constant field (i.e., $BER_{opt.}/BER_{const.}$) according to the different number of molecules per emission N_{EM}.

according to the communication environment, including the detector type, the number of samples per bit interval, the final condition, etc. Optimizing q is recommended for future work.

Fig. 5 shows the ratio of the BERs obtained from adopting the proposed and constant fields (i.e., $BER_{opt.}/BER_{const.}$) according to the different channel radius a_c. This result shows the relative performance between the proposed and constant fields for differently bounded channel environments, where the lower ratio value implies that the proposed field yields a larger BER improvement compared to the constant field. The figure shows that in the cylindrical channel with a large radius (e.g., $a_c = 0.5$ μm), which mimics the unbounded channel, the proposed field without the ISI consideration (i.e., $q = 0$) yields a better BER performance as 75.1% of that of the constant field when $N_{EM} = 300$. It is also noted that the proposed field with $q = 0.1$ generates a 70.7% BER compared to the constant field. However, the ISI consideration (i.e., setting $q = 0.1$) does not significantly improve the BER performance in the channel with a large radius, where the ISI is generally not large because there is enough room for the molecules to diffuse away from the receiver sphere. However, interestingly, the BER ratio of the proposed field without the ISI consideration ($q = 0$) to the constant field suddenly increases as the radius decreases smaller than $a_c = 0.88 \times 10^{-7}$ m. This observation implies that many molecules remain in the receiver sphere (since there is no space for the molecules to diffuse in such a bounded channel) and function as the ISI to hinder the detector from estimating the transmitted bits. In contrast, the proposed field with the ISI consideration $q = 0.1$ could lower the BER by reducing the ISI as designed. This result verifies that the ISI consideration would be a means to avoid the significant ISI deterioration in the bounded duct channel.

Further to the observation above, Fig. 6 shows the ratio of the BERs obtained from adopting the proposed field and the constant field (i.e., $BER_{opt.}/BER_{const.}$) according to the different number of molecules per emission N_{EM}. When N_{EM} is small, it is noted that the gap between the BER ratios without and with the ISI consideration (i.e., $q = 0$ or 0.1) is not significant. In contrast, the gap grows as N_{EM} increases since there are more molecules in the medium that could cause interference as N_{EM} increases. This result indicates that as N_{EM} increases, it is more important to take ISI into account.

V. Conclusion

This paper studied utilizing the electrophoretic force to improve the BER performance in the circular duct channels. Using the calculus of variations, we found the electric field that could quickly place the information-carrying molecules at the receiver site after they are injected into the channel, simultaneously taking the ISI component into account. The numerical results showed that the proposed electric field could significantly improve the BER performance of the MC systems operating in the bounded circular duct channel.

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